Analytical Results of the Los Angeles, CA 2006 1% Manual Tally of Ballots Counted by the MTS Electronic Voting Machines

Brian Dolan
Nasrudin Consulting*
In Response To Dr. Judy Alter

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Abstract

This paper investigates the results of the mandatory 1% manual tally (CA EC 15360) of the June 2006 Primary and Gubernatorial Elections in Los Angeles County observed by Dr. Judy Alter, Director Protect California Ballots. Nasrudin Consulting was asked to impartially describe the data from the manual tally. Two basic questions were (a) was the computer count accurate compared to the hand count and (b) do the errors demonstrate a systematic bias. To accomplish this analysis, we introduce the terms **Gross Errors, Asymmetric Errors, Symmetric Errors** and two types of **Introduced Errors**, along with specific means of calculation. We examine accuracy on two levels: the record and the contest. A "record" is defined as one row of data in the manual tally report. A "contest" is the data for all the candidates for each race in all the precincts that were tallied. We find that the computer count has an accuracy of 81% for each record and 13% at the contest level. A χ^2 test indicates no systematic bias towards any political leaning in the existing errors. A bootstrapping analysis predicts a county election Gross Error rate of $1.0\% \pm 0.2\%$ at the record level, a computer overcount level of $0.3\% \pm 0.1\%$, and a computer undercount level of $0.7\% \pm 0.2\%$.

*www.nasrudinconsulting.com, admin@nasrudinconsulting.com

Contents

1 Introduction

The 2006 Los Angeles County Elections included several controversial measures and contests. The election also incorporated the use of electronic voting machines instead of hand counts. This paper is a reflection of the public interest of the use of electronic voting machines. It is the belief of Nasrudin Consulting that a thorough investigation of the integrity of the electronic voting machines is integral to the safe-guarding of the American democratic system. Investigation with minimal bias and reporting without and agenda is essential to the national debate. We attempt to provide such an investigation and report.

The mandatory 1% Manual Tally is an audit of the 2006 Los Angeles Primary and General Elections. The precincts are chosen by computer randomization under the supervision of the Registrar-Recorder. The tally is done to compare the 1% manual tally against a new hand count. This opens the election up to the scrutiny by the general public.

There are two major questions we hope to address in this paper. The first is whether the computer counts accurately reflect the will of the voters. The second is whether any introduced errors are directional in their political leanings. More to the point: Do the errors in the computer counts favor one party or political ideology over another?

Does the computer count accurately reflect the will of the voters? Votes have been tabulated by hand for hundreds of years, and the notion is deeply embedded within our culture. Recently, our culture has been asked to alter the fundamental mode of democratic elections. A natural concern is consistency between the old and new methods. If the hand count was an acceptable mode for democratic elections, then any new mode must be measured against the hand count. It is for this reason we take the hand count as the "accurate" count. Please see §2.1.

The question of accuracy must be statistically addressed at two distinct levels. For a statistician, it is natural to work at the finest graining allowed–in this case, a single record. However, the popular attention seems to reside at the level of contest, such as all the candidates for governor. "How many contests are accurately tabulated?" is a more natural question than "Is there integrity in each record." We attempt to pursue both paths.

All counting methods are subject to error. When the errors are disproportionately in one direction, the issue of fraud arises. This question assumes the presence of errors and investigates if a bias exists in these errors. If so, what is the bias? We will re-iterate this point, but even an undeniable bias towards one ideology is still just a description of the situation. We can never prove intent, so we can never truly prove fraud, only that, for some reason, all of the errors are in the favor of Party A.

Nasrudin Consulting makes every attempt to remain ideologically agnostic. Our goal is impartially to describe the data as they were delivered to us. While it is true that the questions we ask of the data have been brought to our attention by Dr. Judy Alter and thus incorporate a certain point of view, we have not avoided adjacent questions in our analysis and have allowed ourselves to explore any questions the data posed.

A quick stylistic note. We anticipate a largely non-technical audience for this paper. However, the ideas we are discussing can become very technical. In those instances, we have chosen to avoid the use of academic language. We are striving to be the interface between the data, e.g. counts and error rates, and the information, e.g. the scale of accuracy or inaccuracy of the electronic voting machines. The mathematical analysis is the intermediate step. We wish to illuminate this step without becoming enthralled by it.

Disclaimer

Our mandate is to describe accurately the errors in the election as exposed by the 1% Manual Tally. We offer neither explanation nor accusation to the state of affairs. If violations are of the election code are indicated by this analysis, that is entirely incidental to the descriptive nature of this report.

The nature of the request from Dr. Alter was to examine the flow of votes from 'Conservative' to 'Liberal' and describe any trends in that regard. We make no political statement by our classifications of candidates and measures. Arguments will naturally be generated by our classifications, and we welcome the discussion.

Our data and our actual SQL code will be made available to all interested parties at no cost.

2 Materials and Methods

The data was provided to Dr. Judy Alter in the form of computer print outs from the Los Angeles Registrar-Recorder of Votes in Norwalk of the 1% manual tally from June and November, 2006. These papers were scanned by Barbara Levin and Dan Wang converted the scanned images into text (or "flat" files) using commercial Optical Character Recognition (OCR) software. These files were subsequently cleansed of basic errors using a a mixture of Perl scripting and pattern matching by Brian Dolan. Finally, the data were loaded from the cleansed flat files into a PostgreSQL database maintained by Nasrudin Consulting. Nasrudin Consulting is happy to provide the complete unaltered digital flat files and the Perl scripts involved to any interested parties.

All mathematical analysis was done in R, a statistical programming language provided by The R Project [2].

2.1 Assumptions

Our most fundamental assumption is that the hand count observed by Dr. Alter and other volunteers from Protect California Ballots is the "correct" count. This assumption shapes our language only and has no impact on our statistical analysis. With this assumption, we call differences in the computer count and the hand count an "error" of the computer count. In light of our outcome, reversing this assumption would ultimately suggest that the hand count is grossly inaccurate rather than the computer count.[1]

A further, very important assumption is that the manual recounts are a Simple Random Sample (SRS) of the election results. From this, we postulate that the character of the 1% manual tally is identical, or at least not measurably different from, the character of the county elections. Naturally this assumption will be challenged by other professionals and we welcome the discourse.

2.2 Terminology

We have endeavored to construct an internally consistent set of terms. The nature of the data precludes direct measurement of over or under counting. The next section outlines terms that are mostly used for calculation purposes and are only briefly reported upon. The section following it, §2.2.2, introduces terms that are more immediately recognizable and support the bulk of the analysis.

In this paper, we adopt the convention that zero constitutes a "count". Thus, when we speak of voting records, we generally include a record with 0 votes. This is to accommodate records where candidates were on the ballot but received no votes in a given precinct.

Another shorthand we use is the phrasing "computer votes" or "hand votes" to signify "the numbers of votes counted by computer (hand) and recorded by the Los Angeles County Registrar-Recorder." We beg forgiveness for the abuse of language.

2.2.1 Types of Errors

This section is intended to provide a mathematical underpinning for the investigation, and may be seen as technical. Our hope is that we may introduce

[1] Because our sample set is small, we set our confidence level at $\alpha = 0.01\%$ for the entirety of the analysis. Any test that we identify as "significant" is significant at the 0.01% level.

rigorous terms into the general discussion of the election data.

For purposes of this analysis, we define three types of errors: ‘Gross’, ‘Asymmetric’ and ‘Symmetric’. The ‘Gross’ errors are a simple count, e.g. the sum of the absolute value of the errors. ’Asymmetric Errors’ are the sum of the values and ‘Symmetric’ Errors are Gross - |Asymmetric|. In this example

Candidate	Computer Count	Hand Count	Difference
A	100	101	1
B	100	99	-1

we have 2 Gross errors from $|1|+|-1| = 2$, 0 Asymmetric errors from $1+(-1) = 0$ and 2 Symmetric errors from $2 - |0| = 2$. Changing candidate B, we have the example

Candidate	Computer Count	Hand Count	Difference
A	100	101	1
B	100	98	-2

and we obtain 3 Gross, -1 Asymmetric and $3 - | - 1| = 2$ Symmetric errors.

Each of these notions serve a different purpose. Gross gives us a simple error rate where a vote has been either mis-assigned or there has been an omission or insertion. Symmetric error indicates potential mis-assigment from one candidate to another. Asymmetric error indicates the total ingress/egress of votes.

All fields labled ‘Percent Errors’ are defined under the paradigm

$$100 \times \frac{HandVotes - ComputerVotes}{ComputerVotes} = 100 \times \frac{DiffVotes}{ComputerVotes}.$$

In short, we are trying to normalize to the Computer Votes. This decision is arbitrary, but must be made consistently. When we speak of Percent Gross Error (PGE), we mean

$$100 \times \frac{|HandVotes - ComputerVotes|}{ComputerVotes}.$$

We believe that deliberate fraud would most likely be evidenced by 0 Asymmetric errors and positive Symmetric errors, e.g. votes shifted from one affiliation to another with zero new votes introduced. However, we cannot definitively prove fraud.

2.2.2 Types of Introduced Votes

The above types of errors do not directly address the difference in the hand count versus the computer count. When looking at an individual contest in an individual precinct, it is easy to see which count, hand or computer, had more votes. However, when looking at a precinct across contests, or a contest across precincts, simple sums will not suffice. Below is a simple example. Consider one candidate for an office in two precincts.

Candidate	Precinct	Computer	Hand	Difference
Smith	A	100	99	-1
Smith	B	99	100	1

If we simply added the computer and hand votes, we would detect no errors for this candidate across precincts. In this report, we have the notion of "Computer Added" or "Hand Added" votes. Here, the candidate would have one of each. The "Computer Added" voted coming from precinct A and the "Hand Added" vote coming from precinct B.

Extracting this number from the data requires a small manipulation. We have a column in the data named "DiffVotes" which is defined as

$$DiffVotes = HandVotes - ComputerVotes.$$

When we speak of an "exact match,“ we mean a record or entry that shows $DiffVotes = 0$. This allows us to define the following values.

$$\begin{aligned} Computer\ Added &= \frac{|Sum\ of\ DiffVotes| - Sum\ of\ |DiffVotes|}{2}, \\ Hand\ Added &= \frac{|Sum\ of\ DiffVotes| + Sum\ of\ |DiffVotes|}{2} \end{aligned}$$

One can see that the above values are actually determined in terms of the Asymmetric and Symmetric errors, but the use of those terms would not be elucidating in this context.

It is important to note that the terms "Computer Added" and "Hand Added" are for clarity only and are meant to convey the idea that an additional vote was found during the computer or hand count. These terms do not imply deliberate intention.

2.3 Error Rates

In the absence of an actual error tolerance from the State of California election code, we have chosen a tolerance of 0.5%, or 1 in 200. We have only a subsample of the election data and have only a few hundred data for many of the contests, candidates and precincts we examine. For purposes of this report, we use 0.5% as the maximum threshold.

The Federal Election Commission states an informal, voluntary error threshold of 1 in 300,000, or 0.0003%[3].

2.4 Record vs. Contest Level Errors

The maximum level of detail we can obtain is the number of votes for each candidate on each contest at each precinct. This is considered the "graining" of the sample. Another graining we use frequently is the "contest" level, which is the taken over all candidates and precincts germaine to that contest. To clarify the use of the levels, consider the following data.

Contest	Name	Precinct	Computer Votes	Hand Votes
President	Lincoln	New York	100	101
President	Grant	New York	5	5
President	Lincoln	Georgia	3	3
President	Grant	Georgia	200	200

Here, we see 1 error in 4 for the records (the first) which is 25%. From a contest level, we have a 100% error rate, since 1 of 1 contests showed an error. Collecting by Precinct, we would have a 50% error, since only New York shows an error.

An example of the actual data is given in Appendix C.

3 Analysis and Discussion

In the above §2.3 we describe our determination for an error tolerance of 0.5%. To determine this granularity, we need each event or grouping considered to have at least 200 votes. The majority of our analysis is done on clusters of two hundred or more votes. We note otherwise.

3.1 Detailed Record Analysis and Overall Integrity

Our data contains detail on 8,869 entries at the candidate for each contest at each precinct, covering both the Primary and General elections. That is, we have 8,869 records which tell us how many votes each candidate received at each precinct in both the computer and hand counts.

We note again that a "count" includes 0 votes, which occurs 1,071 times, or about 12%.

These counts match exactly 6,841 (77%) times. Thus, approximately 23% of the time, the hand and computer votes are different. In detail, we have

	Primary	General Election	Total
Records	4,816	4,053	6,841
Exact Matches	3,948	2,893	6,841
Accuracy	81%	71%	77%
Computer Votes	83,179	335,832	419,011
Hand Votes	83,428	337,118	420,546
Gross Errors	1,235	1,880	3,115
PGE	1.48%	0.56%	0.74%
Asymmetric Errors	249	1,286	1,535
PAE	0.30%	0.38%	0.37%
Symmetric Errors	986	594	1,580
PSE	1.19%	0.18%	0.38%

In example, the Percent Gross Errors for the Primary Election would be

$$\begin{aligned} PGE &= 100 \times \frac{|Computer\ Votes - Gross\ Errors|}{Computer\ Votes} \\ &= 100 \times \frac{|419,011 - 3,115|}{419,011} \\ &\approx 1.48\% \end{aligned}$$

3.2 Political Bias in Computer Added and Hand Added Errors

We search for bias at the record level. In the Primary, contests are held between candidates of similar leaning, so bias would require an extremely fine analysis. In the General Election, the contest itself generally has no leaning, except where measures are concerned.

Any "error" we identify necessarily means one count was higher than the other. Since our paradigm is that the hand count is the more accurate count, we consider more computer votes to be an over count and fewer computer votes to be an under count. For clarity, we use the terms "Computer Added" and "Hand Added", though the actual cause of the error is unknown to us. Please also refer to §2.2.2

This analysis was requested out of concern for inaccuracy and fraud within the elections. One concern is whether votes were added towards more Left or Right leaning candidates. In response, we offer the Computer and Hand Added counts now broken out in finer detail.

	Leaning	Computer Added	Hand Added
Primary	Right	129	160
		(0.67%)	(0.84%)
	Left	333	613
		(0.52%)	(0.96%)
General Election	Neutral	41	283
		(0.10%)	(0.70%)
	Right	119	539
		(0.10%)	(0.46%)
	Left	137	761
		(0.08%)	(0.43%)

To interpret this, we need to normalize to the number of votes cast. For instance, if 1,000 votes were cast for the Left and 500 votes were cast for the right, a fair distribution of Computer Added votes might be 10 and 5, or 1% of each. If they were instead 11 and 4, we might infer a bias. The method we use to test this is the *Log-Likelihood Ratio Test* described in Appendix A.

To read the table below, note that the Observed (%) is the observed percent of errors that should go to one particular leaning. In our example above, it would be 1%.

Statistical significance is strongly influenced by sample size. In some cases below, the percentages seem to have a large difference, but the sample size is too small to detect significance. In the first box, we detail the Primary elections. There were 74 Computer Added Votes. We would expect 70% (or about 51) of the Computer Added votes to go to the Left leaning, since the Left leaning accumulated 70% of the overall votes in the Primary. However, we recorded 48 votes, about 65%. Accounting for the small number of observed votes, we cannot call this difference significant.

	Leaning	Observed	Observed (%)	Expected (%)	$\chi^2_{df=1}$
Primary	Left	48	65%	70%	
	Neutral	0	0%	0%	
	Right	26	35%	30%	0.9
General Election	Left	32	46%	45%	
	Neutral	10	14%	10%	
	Right	28	45%	40%	1.8
Overall	Left	80	56%	63%	
	Neutral	10	7%	3%	
	Right	237	38%	34%	8.2

For significance, we need $\chi^2 > 6.64$. Only the Overall Log-Likelihood Test proved significant. It is clear from the table, however, that this is due to the Neutrals being over-represented in the errors. Controlling for this, we get $\chi^2 =$

1.5, which is still not large enough. If we confine ourselves to just the State Measures, we obtain a $chi^{=}0.89$, which is not significant.

In all cuts of the data, whether by contest or candidate, the issue of undercounting is always more severe. We do not offer a specific quantification of the term "severe" except to not that the relationship is sub-linear. We are not able to identify a political pattern, however. We do not see a political bias in the errors from this data.

3.3 The Contests

The data contained 76 General Election contests and 156 Primary contests. Restricting to contests with more than 200 votes, we have 72 General Election and 69 Primary contests.

Of the 141 contests with 200 or more votes, 123 or 87% had discrepancies between the hand and computer counts. All of these precincts have a PGE of greater than 0.1%, or 1 in 1,000. The largest discrepancy was 142 votes for a single contest.

There are 232 contests. Of these, 141 had more than 200 votes associated with the contest[2]. From that pool of 141, we have a median Percent Gross Error (PGE) of 0.71% and a mean PGE of 1.0%. Of the 141 contests, 88 showed a PGE of greater than 0.5%. That means 62% of the contests with more than 200 associated votes showed error rates greater than tolerated by the State of California.

The 88 high-error contests were 29 General Election contests and 59 Primary contests. Of the party primaries, 44 were Democratic and 15 were Republican.

The Primary elections appear much more prone to error than the General elections. There were 78 Democratic and 57 Republican primaries in the raw data. These figures are 54 Democratic and 15 Republican when restricting to vote counts over 200.

The maximum PGE was 5.5% occurring in the contest Country Central Committee, 42^{nd} District, Democratic Primary. A natural break occurs in the PGE data at a PGE of 4.0%. There are six records above 4.0%, all of which are Country Central Committee primary contests for the Democratic party. These are the 40^{th}, 42^{nd}, 45^{th}, 47^{th}, 50^{th} and 58^{th} districts.

[2] A complete list is available from the author.

3.4 Altered Outcomes

An important question is whether the errors involved could lead to different outcomes in any of the contests. We do not have the complete data set for the elections, so the observations below are based premised on the scenario that the entire election would skew in exactly the same way as the sub-sample has.

In this data set, none of the state measures were reversed. The smallest margin of victory was in State Measure 86. According to the computer count was 4.8%, and this margin was maintained by the computer count.

The largest change in margin was for State Measure 84. Under the Computer count, the margin of victory was 23.15% and under the Hand count it was 23.31% for a change in margin of -0.16%[3]. Therefore, the margins of victory were not impacted.

We found no reversals in the winner in the general election. In the primary election, we found four instances when the results would have been altered. These are detailed below.

County Central Committee 39th District Democratic Primary

The Computer decision shows Daniel Delgadillo as the winner in precinct 9000709A/3704 with 45 votes. Second place is given to Ruben Zaragoza with 44 votes. The Hand count show Zaragoza with 45 votes as well, creating a tie.

Of 337 total computer votes, 3 were shown to be have errors for a PGE of 0.89%, above the stated threshold.

County Central Committee 42nd District Republican Primary

The Computer decision has Gary A. Aminoff as the winner of this contest with 51 votes. Mary A. Toman trails Aminoff by one vote for 50, placing her second. When Aminoff loses one vote in the Hand count in precinct 9002230B/4425, he now ties Toman for first.

Here 3 of 356 computer votes had errors for a PGE of 0.84%, above the stated threshold.

County Central Committee 43rd District Democratic Primary

Here, another tie is created between T. P. O. Shaughnessy and Sharon Molander when Molander loses one vote to the Hand count in precinct 90000251A/3422.

[3]Our level of precision has temporarily increased for purposes of illustration

The pair now tie at 23.

1/280 errors lead to a PGE of 0.35%, within the stated threshold.

County Central Committee 55th District Republican Primary

In this instance, a tie is broken between Victor L. Dennis and G. W. Slaugter. Dennis gains a vote in precinct 3450064B/1416 which gives him 27 votes. Slaughter maintains his 26 votes.

This contest contained 7 errors of 236, for a PGE of 2.97%, beyond the stated threshold.

3.5 Close Lead Margins

In consideration of the percent gross errors, we have found 95 instances of lead margins between candidates which were less than the PGE. None of these altered the ultimate winner.

We found 6 instances when the winner held a lead margin over his nearest competitor which was smaller than the PGE. All of these have more than 200 total votes. Of these, 4 were Democratic primaries and 2 were Republican primaries.

Of the 691 candidates and contests, we have 288, or 42% where the lead margins where less than 0.5%. We considered the differentiating of 2nd and 3rd or 5th vs. 6th place, not just 1st versus 2nd. That is, 42% of the leads were not determined to an acceptable tolerance.

3.6 Extrapolation To County Elections

As mentioned in §2.1, our entire analysis is predicated on the idea that we have a Simple Random Sample of the election data. Though the voting population of Los Angeles may show a different political tendency, we assume the error skew is not different in Los Angeles than in other areas. We are examining the behavior of the machine, not the voter, and thus we expect the electronic voting machine to not significantly change its behavior from precinct to precinct or state to state.

In light of this assumption, we performed a rudimentary bootstrap analysis to infer the PGE of the country elections. For details on this method please see Appendix B.

The bootstrap yielded a predicted PGE of $1.0\% \pm 0.2\%$ with a standard error of $2.0\% \pm 0.4\%$. Plainly, we expect 1 in 100 votes tabulated by the computer to be incorrect, plus or minus a fifth of one vote.

To the issue of computer added, votes, the bootstrap method shows a Computer Added percentage of $0.3\% \pm 0.1\%$ with a standard error of $1.2\% \pm 0.4\%$. That is, we expect 3 in every 1,000 votes to be added by the computer.

In the same fashion, we analyzed hand added (or "under count") errors. For the county elections, we see an under count of $0.7\% \pm 0.2\%$, with a standard error of $1.6\% \pm 0.4\%$. We expect that 7 of every 1,000 votes will go uncounted.

3.7 The Individual Voter

At this point in the analysis, we become interested in how it affects an individual voter. This section is done on a heuristic level and is not included in the conclusions. To determine the number of voters we reasoned that any voter who goes to his precinct is likely to vote in the most popular contest of that precinct. We find the most popular contest per precinct and count the number of hand votes in that contest. We infer that number to be the number of voters in that precinct for that election, denoted $\#Voters$.

Next we define and calculate some metrics for an individual voter.

$$\begin{aligned} VotesPerVoter &= \frac{Sum(HandVotes)}{\#Voters} \\ &= \frac{420,546}{18,801} \\ &\approx 22.3, \\ ErrorsPerElection &= VotesPerVoter \times PGE \\ &\approx 22.3 \times 0.74\% \\ &\approx .17. \end{aligned}$$

So a given voter will cast about 22 votes every election. These votes will result in an error about 0.74% of the time and that individual will have about one-fifth of one of his votes incorrectly counted. Generally, for every five elections a citizen participates in, he will have one vote mis-counted.

4 Conclusions

We set out to answer two different questions, one of accuracy and the other of intent. As error rates of 0.1% were common in all cuts of the data, we do not feel a sufficient level of accuracy has been obtained in the computer counts. The State of California has not provided us with a maximum tolerable error, but our assumption is that it will be lower than 0.1%. With the precincts being able to correctly count only 3% of the time, a serious investigation into our election procedures is warranted.

To the question of intent, we could find no evidence of directional bias. Speaking plainly, with the tremendous amount of error in the simple counting, even deliberate and malicious fraud would likely be obscured by the noise of errors.

The data provided by the Los Angeles Country Registrar-Recorder of Votes showed that the computer count was accurate only 81% of the time at the record level. We may expect 19 of every 100 records provided to be inaccurate. This is an appalling level if error.

The contests are much more shocking. Fully 87% of the contests had errors in them.

A Likelihood Ratio Test

A Likelihood Ratio (**LR**) Test provides us with insight at to how likely our results are under the null hypothesis. For instance, if we are given a coin we are told is fair, we may flip it 100 times and expect about 50 H and 50 T. If our actual data **X** is 46 H and 54 T, do we consider this coin biased? An LR test can help us answer this question.

The exact calculation in this case would be

$$\begin{aligned} LR &= \frac{\binom{100}{46}\left(\frac{50}{100}\right)^{46}\left(\frac{50}{100}\right)^{54}}{\binom{100}{46}\left(\frac{46}{100}\right)^{46}\left(\frac{54}{100}\right)^{54}} \\ &= \frac{\left(\frac{50}{100}\right)^{46}\left(\frac{50}{100}\right)^{54}}{\left(\frac{46}{100}\right)^{46}\left(\frac{54}{100}\right)^{54}}. \end{aligned}$$

To use the values, we transform the above equation to make it a **Log-Likelihood Ratio**

$$\begin{aligned} \log LR^2 &= \log\left[\frac{\left(\frac{50}{100}\right)^{46}\left(\frac{50}{100}\right)^{54}}{\left(\frac{46}{100}\right)^{46}\left(\frac{54}{100}\right)^{54}}\right]^2 \\ &= 2\times\log\left[\left(\frac{50}{100}\right)^{46}\left(\frac{50}{100}\right)^{54}\right] \\ &\quad -2\times\log\left[\left(\frac{46}{100}\right)^{46}\left(\frac{54}{100}\right)^{54}\right] \\ &\approx 0.64. \end{aligned}$$

We can look up the value 0.64 in a χ^2 table to see if the result is statistically significant. If it is, we can reject the null hypothesis, in this case the the coin is unbiased. Our value of 0.64 does not return as significant, so we cannot reject the null.

B Bootstrapping

Bootstrapping is a statistical method for extrapolating from a sub-population to the larger population. For instance, we have a sub-population of the votes in the 2006 Los Angeles Election and we want to draw conclusions about the whole population of Los Angeles County Elections.

Rather than a mathematical explanation, we offer the method as an anecdote. Interested readers are encouraged to follow [1].

Suppose we are interested in the average height of the North American population. To that end, we assemble $N = 5,000$ people randomly selected from across the country into a large hall. We, the researches, stand outside the hall and randomly pick $S = 100$ names from a hat and ask them to step outside. Once the group arrives, we take the average height and the standard deviation of the height. We do this $R = 1,000$ times, allowing a person to come out any number of times if they are called.

At the end of the day, we have created 1,000 tests and we have 1,000 average and 1,000 standard deviations. Although it is very likely that everyone in the hall is called at least once, the very very tall and the very very short will show up in only a few of the groups led outside. In that way, their impact on the overall study is minimized.

We now take our 1,000 averages and standard deviations and take *their* average and standard deviation. From one experiment, 5,000 people, we have created 1,000 experiments. The averages themselves should then be "Normally" distributed, meaning they follow a bell curve.

For our purposes, we are examining the "Percent Gross Errors" (PGE) of the election data at the record level, e.g. level of polling location, contest and candidate. We eliminated all instances where there were fewer than 10 computer votes, leaving us with $N = 5,008$ rows of data[4]. We created samples of size $S = 100$ rows and repeated $R = 1,000$ times.

[4] The original data was 8,869 rows long.

C Example Page

Here we provide examples of the pages provided by the Los Angeles County Registrar-Recorder. Each line constitutes one “record” in our terminology. Thus, the next page contains 32 records over 5 precincts with six candidates and one contest. The same is true for the page following that, but with six different precincts.

1% MANUAL TALLY RECAP SUMMARY
GENERAL ELECTION – NOVEMBER 7, 2006

GOVERNOR

PRECINCT NO. (SERIAL NO.)	BALLOTS CAST COMPUTER COUNT	HAND COUNT	DIFF.	CANDIDATE OR MEASURE	VOTES CAST COMPUTER COUNT	HAND COUNT	DIFF.
0150023A/0045	373	372	–1	ART OLIVIER	6	6	0
				JANICE JORDAN	2	2	0
				PETER M. CAMEJO	15	15	0
				EDWARD C. NOONAN	0	0	0
				PHIL ANGELIDES	198	198	0
				A. SCHWARZENEGGER	132	131	–1
0200020A/0088	450	449	–1	ART OLIVIER	10	10	0
				JANICE JORDAN	5	5	0
				PETER M. CAMEJO	15	15	0
				EDWARD C. NOONAN	2	2	0
				PHIL ANGELIDES	261	265	+4
				A. SCHWARZENEGGER	134	135	+1
0350023A/0118	201	201	0	ART OLIVIER	2	2	0
				JANICE JORDAN	3	3	0
				PETER M. CAMEJO	6	6	0
				EDWARD C. NOONAN	4	4	0
				PHIL ANGELIDES	79	81	+2
				A. SCHWARZENEGGER	98	99	+1
0350030A/0122	248	248	0	ART OLIVIER	4	4	0
				JANICE JORDAN	1	1	0
				PETER M. CAMEJO	4	4	0
				EDWARD C. NOONAN	0	0	0
				PHIL ANGELIDES	80	80	0
				A. SCHWARZENEGGER	153	153	0
0750033A/0244	296	296	0	ART OLIVIER	7	7	0
				JANICE JORDAN	4	4	0
				PETER M. CAMEJO	4	4	0
				EDWARD C. NOONAN	2	2	0
				PHIL ANGELIDES	116	116	0
				A. SCHWARZENEGGER	158	159	+1

1% MANUAL TALLY RECAP SUMMARY
GENERAL ELECTION – NOVEMBER 7, 2006

GOVERNOR

PRECINCT NO. (SERIAL NO.)	BALLOTS CAST COMPUTER COUNT	HAND COUNT	DIFF.	CANDIDATE OR MEASURE	VOTES CAST COMPUTER COUNT	HAND COUNT	DIFF.
0750041A/0251	313	313	0	ART OLIVIER	6	6	0
				JANICE JORDAN	2	2	0
				PETER M. CAMEJO	2	2	0
				EDWARD C. NOONAN	6	6	0
				PHIL ANGELIDES	107	106	–1
				A. SCHWARZENEGGER	184	186	+2
1600014A/0647	197	197	0	ART OLIVIER	3	3	0
				JANICE JORDAN	1	1	0
				PETER M. CAMEJO	4	4	0
				EDWARD C. NOONAN	4	4	0
				PHIL ANGELIDES	127	127	0
				A. SCHWARZENEGGER	53	53	0
1960032A/0804	269	269	0	ART OLIVIER	0	0	0
				JANICE JORDAN	2	2	0
				PETER M. CAMEJO	1	1	0
				EDWARD C. NOONAN	3	3	0
				PHIL ANGELIDES	223	225	+2
				A. SCHWARZENEGGER	29	27	–2
2100106A/0880	282	282	0	ART OLIVIER	4	4	0
				JANICE JORDAN	3	3	0
				PETER M. CAMEJO	5	5	0
				EDWARD C. NOONAN	0	0	0
				PHIL ANGELIDES	156	156	0
				A. SCHWARZENEGGER	106	106	0
2300030A/0913	254	254	0	ART OLIVIER	0	0	0
				JANICE JORDAN	4	4	0
				PETER M. CAMEJO	6	6	0
				EDWARD C. NOONAN	5	5	0
				PHIL ANGELIDES	214	214	0
				A. SCHWARZENEGGER	18	18	0

References

[1] W.N. Venables and B.D. Ripley. Modern Applied Statistics With S. New York, Springer. 2003.

[2] The R Project for Statistical Computing, www.r-project.org.

[3] Federal Election Commission